Périodes, motifs et équations différentielles : entre arithmétique et géométrie

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Anabelian representations of the motivic Galois group

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Given a variety X over a field k embedded into $\mathbf{C}$, the motivic Galois group acts on a quotient of the algebraic completion of the fundamental groupoid of $X(\mathbf{C})$. We will discuss a motivic version of a theorem of Pop characterising the motivic Galois group via these actions.

André's flatness lemma and applications

Bhargav Bhatt [1]

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A key innovation in André's solution of the direct summand conjecture was a certain flatness lemma for perfectoid rings. In this talk, I will recall this lemma and its relevance for commutative algebra. I will then explain how the lemma surprisingly also provided the essential ingredient in the proof of the odd vanishing theorem in the algebraic K-theory of p-adically complete rings. Joint work with Peter Scholze.

Degeneration loci of l-adic local systems

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I will make a partial survey of what is expected and known about the degeneration loci of l-adic local systems over varieties over number fields. For those arising from geometry, the degeneration loci can be interpreted in terms of fibers carrying exceptional Tate cycles.

Rational and holomorphic sections of abelian schemes

Pietro Corvaja [1]

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Given an abelian scheme A→B over an algebraic base B, its Mordell-Weil group is the group of rational sections $B \to A$; it is known to be discrete. In a joint work with J. Noguchi and U. Zannier, we consider instead the group of holomorphic, possibly transcendetal, sections. We prove some kinds of 'Big Picard' results, showing e.g. that transcedental sections cannot omit relatively ample divisors.
As a by-product, we introduce a Nevanlinna analogue of the theory of heights for transcendental sections, developing the theory of the Betti maps, previously investigated together with Y. André and U. Zannier.

E-functions and geometry

Javier Fresán [1]

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Among the contributions of Yves André that have been a great source of inspiration for me over the years are his book "G-functions and geometry", where he proves for example that solutions of differential equations of geometric origin over the field of algebraic numbers are G-functions, and his two papers "Séries Gevrey de type arithmétique", where he determines the structure of the differential equations satisfied by E-functions and deduces a new proof of the Siegel-Shidlovsky theorem. I will survey on past and ongoing work with Peter Jossen in which we build on these fantastic results to study E-functions from the point of view of exponential motives. As a byproduct, we were for example able to answer in the negative Siegel's question whether all E-functions are polynomial expressions in hypergometric functions.

New perspectives on de Rham cohomology, after Bhatt-Lurie, Drinfeld, et al.

Luc Illusie [1]

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I will discuss some of the new developments on de Rham cohomology in mixed characteristic due to Bhatt-Lurie, Drinfeld, et al., with special emphasis on the hidden structures discovered on de Rham cohomology in positive characteristic in presence of liftings.

Limits of complex integrals

François Loeser [1]

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I will explain how non-archimedean integrals considered by Chambert–Loir and Ducros naturally arise in asymptotics of families of complex integrals. To perform this analysis we work over a non-standard model of the field of complex numbers, which is endowed at the same time with an archimedean and a non-archimedean norm. Our main result states the existence of a natural morphism between bicomplexes of archimedean and non-archimedean forms which is compatible with integration. This is joint work with Antoine Ducros and Ehud Hrushovski.

Gushel-Mukai varieties in characteristic p

Ben Moonen [1,2]

[1] Institute for Mathematics, Astrophysics and Particle Physics – Netherlands
[2] Radboud university [Nijmegen] – Netherlands

I will try to explain some of the main ideas in joint work with Lie FU (Nijmegen), in which we prove the Tate conjecture for 6-dimensional Gushel-Mukai varieties over finitely generated fields of characteristic $p > 5$. This is based on the strategy employed by Madapusi Pera in the case of $K3$ surfaces, and it uses the beautiful work of Yves André on (families of) varieties with a motive of $K3$ type. In order to carry out this programme, we need several basic results about Gushel-Mukai varieties in positive characteristic, and some of these pose an interesting challenge. For instance, our proof that these Gushel-Mukai varieties have no nonzero global vector fields is a tour de force, which relies on computer algebra, and we are unable to extend the main result to $p = 5$ only because these calculations get out of control.

Motives, groups and vector bundles

Peter O'sullivan [1]

[1] Australian National University – Australia

One of the main difficulties in the theory of pure motives is that, in the absence of a proof of the standard conjectures, it is not known that the category of motives modulo homological equivalence is abelian. Yves Andre has shown a way to circumvent this difficulty by passing to a category of motives defined using the larger class of motivated cycles instead of algebraic cycles modulo homological equivalence. We describe another approach to the problem, using an analogy between motives and equivariant vector bundles.

The Hecke Orbit conjecture

Frans Oort [1]

[1] Utrecht University – Netherlands

We report on joint work with Ching-Li Chai.
The *isogeny class* of a polarized abelian variety (A, μ) with moduli point $x = [(A,\mu)]$ is denoted by $H(x)$, called the *Hecke orbit* of x. What is the Zariski closure of the set $H(x)$? In characteristic zero any Hecke orbit is everywhere dense in the moduli scheme of polarized abelian varieties. We easily see that this is not true in general (in the non-ordinary case) in positive characteristic. We consider and prove:

Conjecture / Theorem **(HO).** *The Hecke orbit $H([(A,\mu)])$ is everywhere dense in the NP* stratum given by $\xi = N(A)$.

This was proved by Ching-Li Chai for *ordinary* abelian varieties (Invent. Math. 1995) and formulated as a conjecture in the general case (FO, 1995).

In order to obtain a proof, as a starting point, properties of (moduli of) *supersingular abelian varieties* were established. Several (new) concepts and their properties were needed:

- *stratifications* (EO and NP),
- *foliations* (central leaves, isogeny leaves),
- minimal p-divisible groups, hypersymmetric abelian varieties,

- a proof of a conjecture by Grothendieck,

- a generalization of Serre-Tate coordinates,

in order to understand the *geometry* of these moduli spaces in positive characteristic. Many aspects of these had to be established, such as dimension, irreducibility, the graph of strata with partial ordering given by "inclusion in the boundary". This might explain why it took us 25 years to prove the conjecture.

In this talk I will explain these notions and formulate the *basic strategy of approach*: specialization to the "boundary" of various strata. Examples and relevant structure theorems will be amply discussed. Then I will sketch a proof of (HO), divided into two aspects: the discrete and the continuous HO problem.

Point-counting and the André-Oort conjecture

Jonathan Pila [1]

[1] Mathematical Institute, University of Oxford – United Kingdom

This talk will describe the point-counting strategy for the André-Oort conjecture, emphasizing the triple role of point-counting in the full proof recently announced.

Siegel's problem for E-functions

Tanguy Rivoal [1]

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Siegel defined and studied in 1929 the class of E-functions. They are power series with algebraic Taylor coefficients that satisfy certain archimedean and non-archimedean growth conditions, and solutions of linear differential equations with polynomial coefficients. They generalize the exponential function and, more generally, confluent hypergeometric series with rational parameters are E-functions. Siegel asked if hypergeometric functions enable to construct all E-functions (in a precise sense). After recalling several classical number theoretical results on E-functions, I will survey recent results on Siegel's question, that is now known to have a negative answer thanks to Fresan and Jossen. I will then focus on a "conditional" negative answer we had obtained earlier with Stéphane Fischler (Université Paris Saclay).

Hypergeometric mirror maps

Julien Roques [1]

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Mirror maps are power series which occur in mirror symmetry as the inverse for composition of power series of the form $q(z) = \exp(\omega_2(z)/\omega_1(z))$, called canonical coordinates, where $\omega_1(z)$ and $\omega_2(z)$ are particular solutions of the Picard-Fuchs equation associated with certain one-parameter families of Calabi-Yau varieties. In several cases, the mirror maps have integral coefficients. In this talk, we will give an overview of the integrality properties of mirror maps associated to the generalized hypergeometric equations. We will end with some open problems.

Analytic Geometry

Peter Scholze [1]

[1] MPIM Bonn – Germany

(joint with Dustin Clausen) Before the introduction of the general notion of schemes, algebraic geometry essentially dealt with algebraic varieties over a given field k, and coherent sheaves on them. Analytic geometry, concerned with complex-, real-, or p-adic rigid-analytic spaces is in a similar state: Finiteness conditions are critical to resolve the analytic and topological issues involved.

Recently, replacing topological modules by condensed modules, we have been able to define a general "derived category of quasicoherent sheaves" in all of these settings, and moreover replacing topological rings by condensed rings, we have been able to define a category of "analytic spaces" giving an analogue of the category of schemes in the setting of analytic geometry. The resulting theory naturally includes "derived analytic spaces" as well as arithmetic base rings like the subring of $\mathbb{Z}((T))$ of series that converge on some small archimedean disc. We will give an overview of these developments.

Primitives of algebraic functions

Jacob Tsimerman [1]

[1] UNIVERSITY OF TORONTO – Canada

Given an algebraic function , when can a primitive of it be constructed by means of algebraic operations and taking primitives of some other given algebraic functions? When the only primitive allowed is the logarithm, this is the question of elementary integrability and a decision procedure has been given by Risch. We will present a general decision procedure for deciding this question, based on a new result of Ax-Schanuel type for primitives of differential forms on curves.

Generic bounded solutions and maximal slope quotients of overconvergent F-isocrystals on curves

Nobuo Tsuzuki [1]

[1] Tohoku University – Japan

Let $\mathcal{M}^\dagger$ and $\mathcal{N}^\dagger$ be overconvergent F–isocrystals admitting slope filtrations as convergent F–isocrystals on a smooth variety over a perfect field k of charactersitic $p > 0$. K.S.Kedlaya asked a question: if the first step of slope filtration of $\mathcal{M}^\dagger$ is isomorphic to that of $\mathcal{N}^\dagger$, then $\mathcal{M}^\dagger$ and $\mathcal{N}^\dagger$ are isomorphic. In this talk we prove Kedlaya's question in the case of curves (M.D'Addezio has affirmatively solved the problem in general recently). We study generic bounded solutions and their slopes of F–isocrystals, and explain our key notion "PBQ" F–isocrystals on curves. For local objects, this notion was introduced by B.Chiarellotto and the speaker to study Dwork's conjecture on log-growth of solutions.

The p-adic Corlette-Simpson correspondence for abeloids

Annette Werner [1]

[1] Goethe University Frankfurt – Germany

This is joint work with Ben Heuer and Lucas Mann. For an abeloid variety A over a complete algebraically closed field extension K of the p-adic numbers, we construct an equivalence between finite-dimensional continuous K-linear representations of the Tate module and a certain subcategory of the Higgs bundles on A. To do so, our central object of study is the category of vector bundles for the v-topology on the diamond associated to A. We prove that any pro-finite-étale v-vector bundle can be built from pro-finite-étale v-line bundles and unipotent v-bundles.

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